Posthuma Fosteri:

THE
DESCRIPTION
OF A
RULER,

Upon which is inscribed divers

SCALES:
AND
The Uses thereof:

Invented and written by

Mr. *SAMUEL FOSTER,*

Late Professor of ASTRONOMIE in
GRESHAM-COLLEDG.

By which the most usual *Propositions* in
Astronomy, Navigation, and *Dialling,* are facily
performed. Also, a further use of the said *Scales* in
delineating of far declining Dials; and of
those that Decline and Recline
three severall wayes.

With the delineating of all Horizontall Dials, *between*
30 *and* 60 gr. *of* Latitude, *without drawing any lines
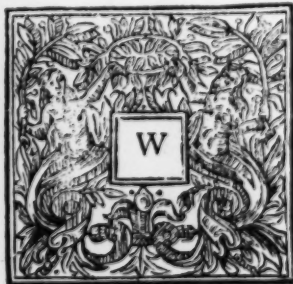but the Houres themselves.*

TO THE
READER.

Courteous Reader,

E here present to thy view , this short Treatise, (written by that learned *Professor* of Astronomie in *Gresham Colledge*, Mr. Samuel

A 2 Foster

FOSTER deceafed) containing in it the *Defcription* and *Ufe* of certain *Lines* to be put upon a ftreight *Ruler*, in the ready folution of many necessary *Queftions*, as well *Geometricall*, as belonging to *Aftronomie*, *Navigation*, and *Dialling*.

We fhould not thus haftily have thruft this into the *World* without its *fellows*, had we not been affuredly informed that fome *people*, greedy rather of *unjuft gain* to themfelves, then with *honefty* to fit ftill, had prepared one for the *Preffe*, from a *fpurious* and *imperfect Copie*, both to the *abufe* of *thee*, and *difcredit* of the *induftrious Author*: who had he thought

                                              fuch

such *things* as these worthy *him* or the *Preſſe*, could have daily *cram'd* thee with *them*, to his own loſſe of *time*, and thy *ſatiety*. However, ſuch as it now is, we aſſure thee was his *own*, and doubt not, but thou wilt finde it *pleaſant* in the *uſe*, *profitable* to *thee*, and *portable* in it *ſelfe*.

We thought fit farther to *adver-tiſe* thee, that there are *abroad* in particular *hands*, *imperfect Copies* of ſome other *Treatiſes* of the ſame *Author*: Namely, An eaſie *Geometricall* way of *Dialling*. Another *moſt eaſie way* to *project houre-lines* upon all kinde of *Superficies*, without reſpect had to their *ſtanding*, either in *reſpect* of *De-clination*

clination or *Inclination.* A *Quadrant* fitted w:th *lines* for the *solution* of moſt *Queſtions* of the *Sphere*: with ſome other *things* of the *like nature.* We fear leaſt *ſiniſter ends* of ſome *mean Artiſts*, or *ignorant Mechanicks*, (for thoſe of *ingenuity* in whoſe *hands* they may be, *we* no way *diſtruſt*) may *engage* them to *father* theſe *things* as *their own*, or at leaſt *under* the *Authors name* put out *lame* and *imperfect Co-pies* of otherwiſe *good things*: To prevent *which* we give *thee* this *timely notice*, aſſuring *thee*, that *theſe*, together with divers *other pieces* never *yet ſeen*, except by *very few*, and if *we deceive not our ſelves*, of much *greater weight*,
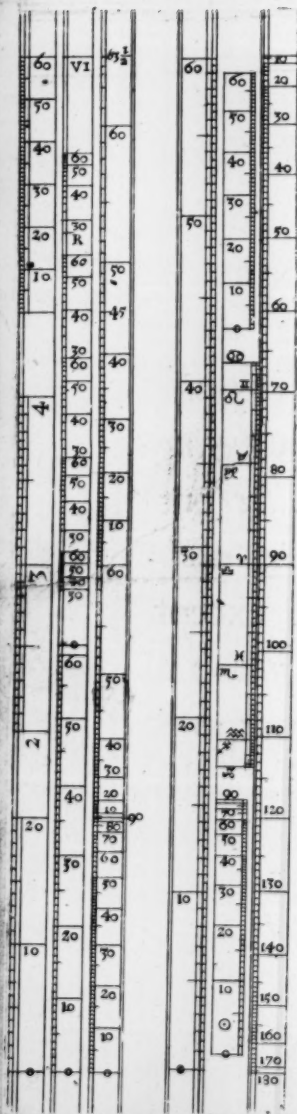
are

are *making ready* for the *Preſſe* by the *Authors approbation*, and from *his own Copie* in *our command*, with his *other papers*, of which *thou* ſhalt be made *partaker* within *few moneths.* In the *mean time*, we deſire *thee* not to *loſe thy time* in *reading*, or *money* in *buying* any the *forementioned Treatiſes* put *out* by *any other*, either under *their own*, or our *Authors name*, except ſuch as ſhall be *atteſted* by *me*, who am *one of thoſe intruſted* for that purpoſe, and who ſhall be *ever ſtudious* of thy good.

Grayes Inne,
July 26, 1652.

Edm. Wingate.

The description of the
Scales on the Ruler.

*T*Here *are 9* Scales *upon the* Ruler.
    1. *Of* Inches *or equall parts.*
    2. *Of* Horizontall spaces : *with*
    3. *A* Scale *of* 60 Chords *fitted to the same Radius, proper to that* Horizontall Scale.
    4. *A* Scale *of* Sines, *to a Radius of two Inches.*
    5. *A* Scale *of* Secants : *and*
    6. *Of* Tangents , *both of the same Radius with the* Sines.

All these are upon one side, On the other side is,

    7. *A little* Scale *of* 60 Chords, *of the same length with the common Radius of* Sines, Secants, Tangents, *and common in use to them all.*
    8. *A large* Scale *of* Versed-Sines *of the whole length of the* Ruler, *with a* Zodiac *annexed to it.*
    9. *A* Scale *of unequall parts divided into* 90, *noted with* ☉, *of two Inches Radius as is the line of* Sines.

THE
Vſes of the SCALES on the
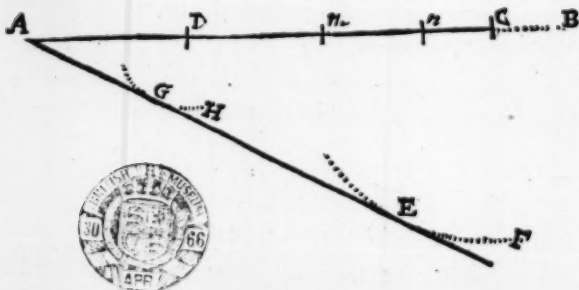# RULER.

CHAP. I.
Of the Scale of equall parts.

He Scale of Inches is a Scale of
equall parts, and will performe
(by protraction upon paper) ſuch
concluſions as are uſually wrought
in Lines and Numbers, as in Maſter
Gunters 10. *Prop.* 2. *Chap. Sector*,
may be ſeen, and in others that have written in the
ſame kinde.

An Example in Numbers like his 10th. *Prop.*

As 15 to 5, So 7 to what?

Here, becauſe the ſecond terme is leſſe than the
                    B   firſt

firſt, upon the line A B, I ſet A C the firſt terme
15, and A D the ſecond terme 5, both taken out of
the Scale of equall parts.   Thence alſo the third
number 7 being taken, with it (upon the center C)
I deſcribe the arke E F, and from A, draw the line
A E, which may only touch that ſame arke. Then
from D, I take D G, the leaſt diſtance from the
line A E, and the ſame meaſured in the ſame Scale
of equall parts, gives 2⅓, the fourth terme re-
quired.

But if your ſecond terme ſhall be greater than
the firſt, then the form of working muſt be chang-
ed; as in this Example.

<center>As 5 to 15, So 20 to what?</center>

Upon the line A B, I ſet the ſecond terme 15,
which here ſuppoſe to be A D: then with the firſt
terme 5, upon the center D, I deſcribe the arke GH,
and draw A G that may juſt touch it.   Again, ha-
ving taken 20 the third term, out of the ſame Scale,
I ſet one foot of that extent upon the line A B, re-
moving it till it fall into ſuch a place, as that the
<div align="right">other</div>

other foot being turned about will juftly touch the
line A G before drawn, and where (upon fuch con-
ditions) it refteth, I make the point C. Then mea-
furing A C upon your Scale, you fhall finde it to
reach 60 parts, which is the fourth number re-
quired.

The form of work (though not fo Geometricall)
is here given becaufe it is more expedite than the
other by drawing parallel lines.   But in fome pra-
ctifes, the other may be ufed.

I have been the more large upon this, becaufe the
folutions of proportions which follow muft be re-
ferred hither, the forme of their operations being the
fame with this.   In them therefore fhall only be
intimated what muft be done in generall, the par-
ticular way of working being here explicated.

CHAP. II.
Of the Scales of Chords.

THe Scales of Chords are to protract and
measure angles. The manner how they muft
be ufed is well enough known.

Only note here, that you may make the line of
Sines, the line of Verfed Sines, or the Zodiacke,
(beginning at the middle of thefe two laft mention-
ed) to ferve for Chords of feverall extents, if you
count each halfe degree for a whole degree, and fo
double all the numbers, accounting 10 to be 20,
and 30 to be 60, 45 to be 90, &c. By this you

are fitted with severall Scales of Chords which are
of different lengths, and may be used, each of them,
as occasion shall require.   And (by the way) the
Versed Scale being taken for Chords , it will be of
the same Radius or length with the Sines, Tangents
and Secants and so will protract Angles to a Circle
of their Radius, which is usefull in Projections, and
many other things : and so the little Scale of 60
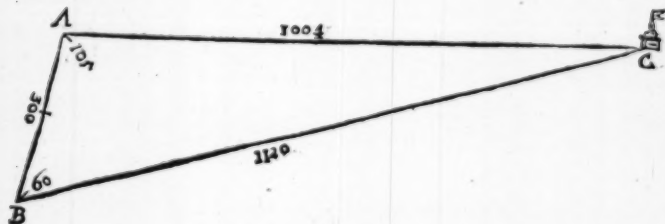Chords might be spared.

CHAP. III.
The joynt use of equall parts and Chords.

B Y these two together , may be resolved all
Cases in Plaine Triangles without proporti-
onall work , if the three quantities given be
protracted by help of these two Scales.   For their
principall uses are to measure lines and angles.

Here must be remembred. First, that if the three
angles alone be given, then will the proportions on-
ly of the sides be found, but not the sides themselves.
Secondly, that if two angles be known, then is the
third also known ; because it is the residue of the
sum of the other two to 180 *gr.*

One case which is of frequent use may here be
given for an example. In mensurations of distances
of places (as Towns or Forts) there are usually two
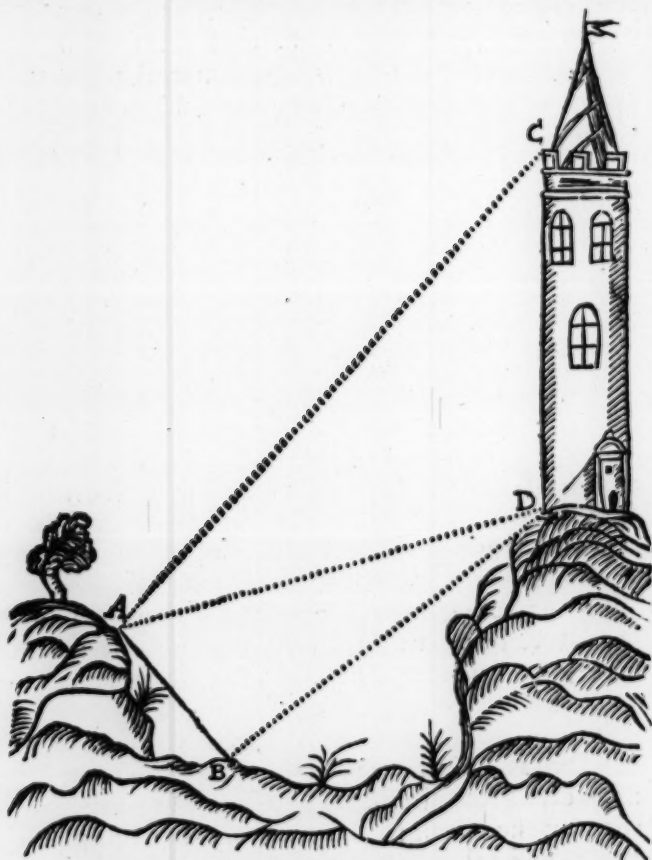stations taken as A and B, whose distance A B
sup-

suppofe known, 300 feet, and the two angles adja-
cent, at A 105 *gr.* at B 60 *gr.* known alfo by ob-
fervation.   To finde the other fides, draw A B, and
upon it fet 300 being taken from the Scale of equall
parts.

Then with a line of Chords protract the angles
at A and B according to their known quantities, fo
fhall the two legges of the fame angles meet at C :
And if the length A C be taken and meafured upon
your Scale of equall parts, the fame will fhew about
1004, intimating that C is from A 1004 feet.   So
again B C being meafured in the fame Scale will
give 1120 ; fhewing that from B to C are about
1120 feet.

In this manner may perpendicular altitudes (as
of Towers or fuch like) be meafured, though no
acceffe can be had to them ; and that much better
than by the Geometricall Square.   For it is not
here requifite that the ground whereon the menfu-
ration is made, fhould be levell , as if you work by
the Square it is moft commonly required ; neither
are you tyed to right angled Triangles here, as there
you are.   As for example,

If

If one station be at A, and the other at B , you
may, by the precedent work get the distance A D.
Then (standing at A) observe the altitude of C, the
complement of that altitude gives the angle A C D.
And

And again from A, if you obferve the altitude of D, the difference of thefe two altitudes obferved, gives the angle C A D. Or if D had appeared lower then your ftation, then the fumme of your two obferved altitudes had made the angle C A D. However, you have now the three angles and fide A D; you may therefore, by help of them, finde the length of C D.

In fuch manner may all cafes in plain *Trigonometry* be refolved.

CHAP. IV.

Of the Scales of Sines, Secants, and Tangents.

THefe being joyntly ufed with a Scale of equall parts, will refolve all things in plain Triangles, by working fuch proportions as are ufually given for that purpofe. The manner of the work may be gathered by the former delineation in the 1 *Chap.* For if A C and A D had been taken out of the Scale of Sines, or Tangents, or Secants; and C E, D G, out of the Scale of equal parts; then had the work been refolved in Sines and equall parts, or Tangents and equall parts, &c. And fo this kinde of work will produce the quantity required; although there be no delineation of the particular parts of the Triangle, as was before done by protraction.   By

By these same three Scales of Sines, Tangents, and Secants, may be wrought all things pertaining to Sphericall Triangles. That is to say.

1. *Proportions in Sines alone.*

2. *Proportions in Tangents alone.*

3. *Proportions in Secants alone.*

4. *Proportions in Sines and Tangents together.* $\begin{cases} \textit{By naturall Sines and Tangents.} \\ \textit{By} \odot \textit{and Versed-Sines.} \end{cases}$

5. *Proportions in Sines and Secants together.*

6. *Proportions in Tangents and Secants together.*

An Example in Sines alone

What Declination shall the Sun have in the 10 gr. of Aries?

Upon the line A B *(see Chap. 1.)* set the Radius or Sine of 90 A C: and make A D equall to the Sine of 10 *gr.* (which is the Suns distance from the next Equinoctiall point.) Then with the Sine of 23½ (taken out of the same Scale of Sines) upon the center C, describe the arke E F; to which, from A, draw the Tangent line A E. Lastly, from D, to this line A E, take the least distance, the same mea-
sured

sured in the line of Sines, gives about 4 gr. for the declination required.

The proportion that is here wrought stands thus.

As the Radius,   to the Sine of 23½;
So the Sine of 10, to the Sine of 4 gr.

The like manner of work is to be used in Sines and Tangents (or any of the other two) joyned together; if it be remembred that the greater terms be kept upon the line A B; as was before prescribed in the first Chapter.

CHAP. V.
OF NAVIGATION.

Some things in this kinde will be performed very conveniently by these lines: As,

Sect. I.

To make a Sea-chart after Mercators *projection.*

A Sea-chart may be made either generall or particular; I call that a generall Sea-chart, whose
C   line

line A E, in the following figure, reprefents the E-quinoctiall, as the line A E there doth the parallel of 50 *gr.* and fo containeth all the parallels fucceffively from the Equinoctiall towards either Pole: but they can never be extended very neere the Pole becaufe the diftances of the parallels increafe fo much, as the Secants doe. But notwithftanding this, it may be termed generall, becaufe that a more generall Chart cannot be contrived in *plano,* except a true Projection of the Sphere it felfe.. And I call that a particular Chart which is made properly for one particular Navigation, as if a man were to fail between the Latitude of 50 and 55 *gr.* and his difference of Longitude were not to exceed 6 *gr.* then a Chart made (as the figure following is) for fuch a Voyage, may be called particular.
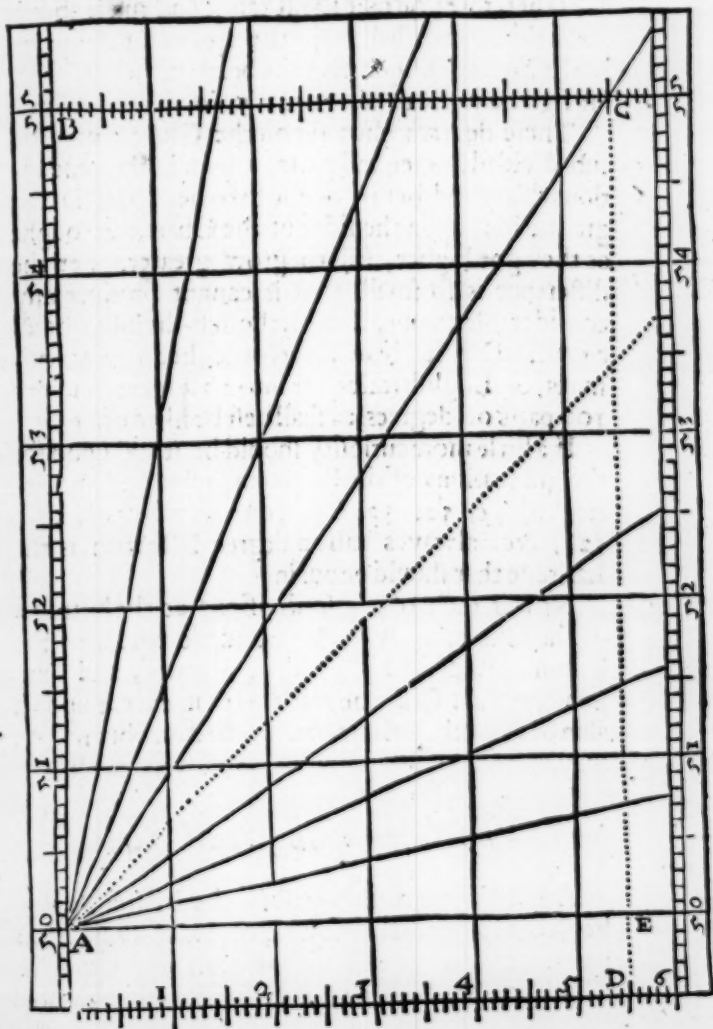
Now the making of fuch a Chart, is Mafter *Gunters* firft propofition page 104 of the Sector, and this the line of Secants will fufficiently perform.

For if it were required to project fuch a Chart: Having drawn the line A B, and having croffed it at right angles with another line A E, reprefenting the parallel of 50 *gr.* you muft then take the Secant of 51 from your Scale, and fet it from 50 to 51 on both fides the Chart, and draw the parallel 51 51.

Again, take the Secant of 52 from your Scale, and fet it upon your Chart from 51 to 52, and fo draw the parallel 52 52. And fo you are to draw the reft of the parallels.

Then for the Meridians, or divifions of the line B C, they are all equall to the Radius.

If

If therefore you take the Radius, and run it above and below, you shall make the spaces or diſtances of the Meridians ſuch as in the bottome of the Chart are figured with 1, 2, 3, 4, 5, 6.

Theſe degrees thus ſet on the Chart, may be ſub-divided into equall parts, which in the graduations above and below ought ſo to be. But in the graduations upon the ſides of the Chart, they ought as they goe higher, ſtill to grow greater. Yet the difference is ſo ſmall that it cannot produce any conſiderable errour, though the ſub-diviſions be all equall. Divide them therefore either into 60 minutes, or Engliſh miles, or into 20 leagues, or into 100 parts of degrees, as ſhall beſt be liked of.

If a little more curioſity ſhould be ſtood upon for the graduations of the Meridian, inſtead of the Secants of 51, 52, 53, &c. you may take $50\frac{1}{2}$, $51\frac{1}{2}$, $52\frac{1}{2}$, &c. alwayes halfe a degree leſſe than is the Latitude that ſhould be put in.

Now if each of thoſe diviſions at the bottome of the Chart, as A 1, &c. be made equall to the common Radius of the Sines, Secants, and Tangents, and if a Chart be made to that extent upon a skin of ſmooth Velame, well paſted on a board, you may work upon it many concluſions very exactly.

The Uſes of the Sea-Chart

Are ſet down in 12 *Propoſitions* by Maſter *Gunter*, beginning *page* 121. In each of which *Propoſitions* is ſhewed how to reſolve the Queſtion upon the Chart it ſelfe, which will be direction enough how

the

the work muſt be performed, without any more words here uſed.

The working of theſe propoſitions alſo may be applyed to the Scales of Sines and Tangents, on the Ruler, and wrought by protraction, according to the rules given in the firſt Chapter, if the proportions, as he layes them down in the foreciced pages, be ſo applyed.

If a Scale of Rumbs be thought more expedient for theſe operations then is a Scale of Chords, it may be put into ſome ſpare place of the Ruler.

His two Propoſitions, page 114. 116, may be done upon the Chart as is there ſhewed, but his ſecond Propoſition, which is,

Sect. 2.

To finde how many Leagues doe anſwer to one degree of Longitude, in every ſeverall Latitude.

This (I ſay) may be done upon the Scales of Sines and equall parts: And for this purpoſe, the two laſt inches of the ſame Scale of equall parts, being equall in length to the Radius or Sine of 90, are divided into 20 at one end, and into 60 at the other end.

Take therefore upon the line of Sines, the complement of the parallels diſtance from the Equator, (or the complement of the given Latitude) and meaſuring it upon the Scale of 20 parts, it will ſhew
you

you what number of Leagues make one degree of Longitude in that parallel of Latitude. And being meafured upon the Scale of 60 parts, it gives fo many of our miles, or fo many minutes of the Equinoctiall, or any other great circle, as are anfwerable to one degree of Longitude in that Latitude.

Example,

Let it be required to finde how many Leagues doe anfwer to one degree of Longitude, in the Latitude of 18 *gr.* 12′.

Take out of the line of Sines, the complement of the given Latitude, namely. 71 *gr.* 48′. Then applying this diſtance to the Scale of 20 equall parts, you ſhall finde it to reach 19, and fo many Leagues doe anfwer to one degree of Longitude, in the Latitude of 18 *gr.* 12′.

And the fame diſtance being meafured upon the Scale of 60 equall parts, will give you 57 parts, and fo many minutes of the Equator are anfwerable to one degree of Longitude, in that parallel of Latitude.

So likewife, in the Latitude of 25 *gr.* 15′, if you take the complement thereof 64 *gr.* 45′, out of the Scale of Sines, and apply it to the former line of 20, you ſhall finde it to reach 18 parts, and fo many Leagues doe anfwer to one degree of Longitude, in the Latitude of 25 *gr.* 15′.

⟪ In the Appendix to Maſter *Norwoods* Doctrine of Triangles, there is by him laid down

down 15 Queſtions of ſailing by the plain
Sea-chart, and others by *Mercators* Chart,
all which the line of Chords and equall parts
will ſufficiently perform, if the work of the
third Chapter of this Booke be rightly un-
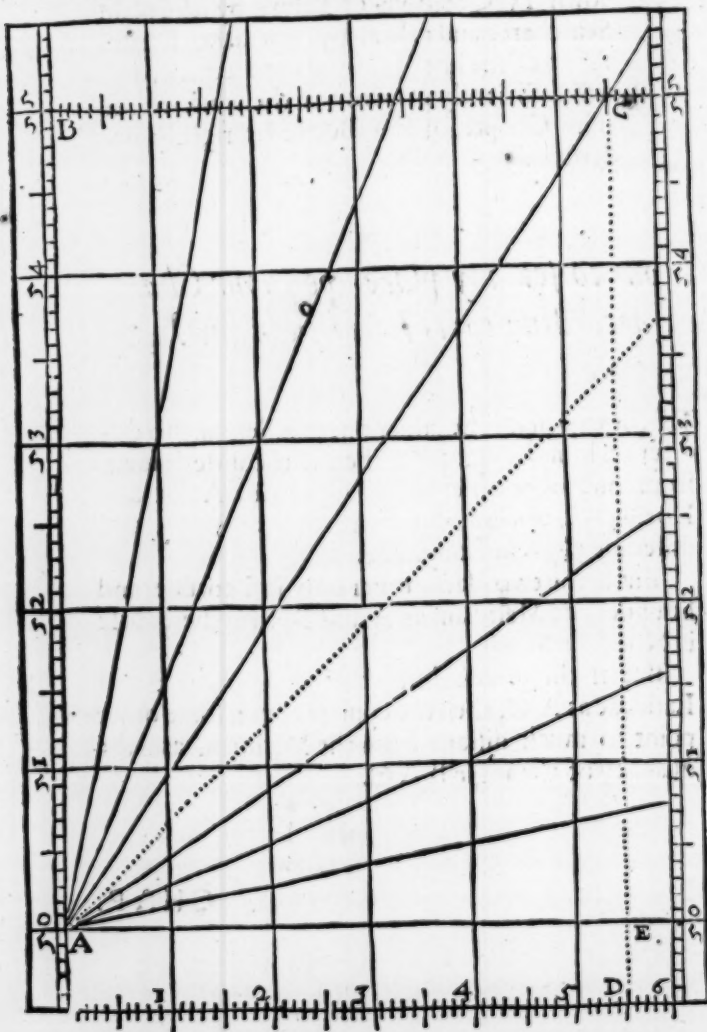derſtood.

Sᴇᴄᴛ. 3.

How to ſet any place upon your Chart, according to its Longitude and Latitude.

IF the two places lie under one parallel, and ſo dif-
fer only in Longitude, then the courſe leading
from one to the other is Eaſt or Weſt : As A and
E being two places under the parallel of 50 *gr.* and
differing 5½ *gr.* in Longitude.

But if the two places differ only in Latitude, and
lie under one Meridian, as A and B, then the courſe
is North or South.

But if the places differ both in Longitude and
Latitude as A C, then the courſe is upon ſome other
point ſo much diſtant from the Meridian, as is the
quantity of the angle B A C.

CHAP.

CHAP. VI.

Of Projections of the Sphere.

FOr this purpose chiefely, is the lesser line of Chords added, being made to the same Radi-us that belongs to the Sines, Secants, and Tangents. For when any Projection is to be made, the fundamentall Circle must be of that common Radius, and then the angles to be inserted upon it may be taken out of this line of Chords which is fitted to it. See the second Book of the Sector, *Chap.*3. For these Tangents and Secants will per-forme the same things in those *Stereographicke* pro-jections that there are done ; and in all other irregu-lar projections likewise.

By this kinde of work may any Sphericall con-clusions be performed by protraction in *plano.* Also true Schemes of the Sphere may be drawn, sutable to any question, which will not a little direct in Sphe-ricall calculations.

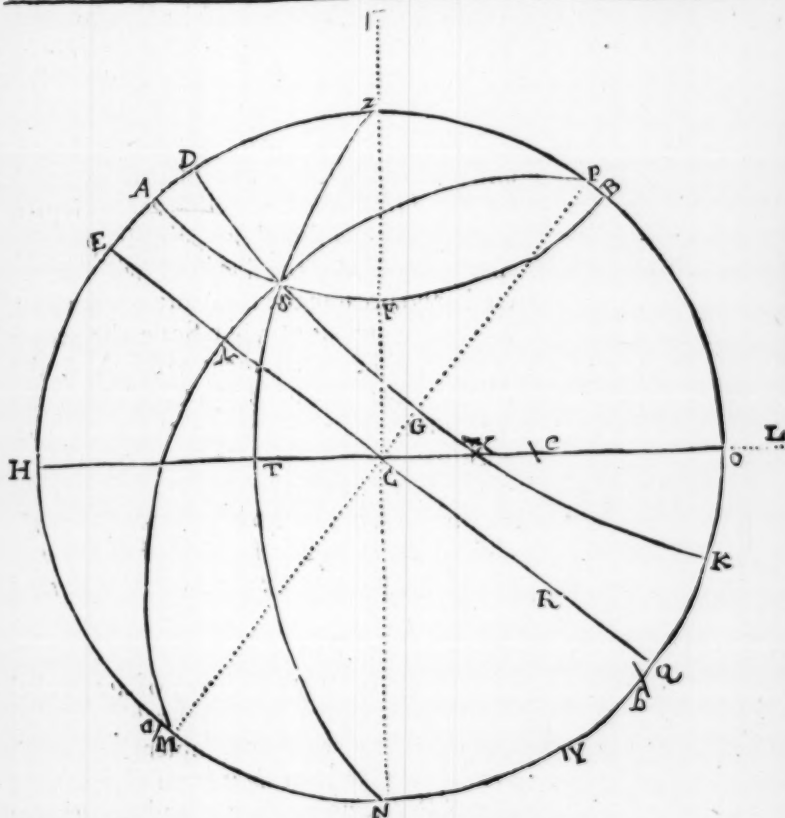As suppose it were required to project the Sphere sutable to this Question.

Having the Latitude of the place, the declination of the Sun, and the Altitude of the Sun, to finde either the Azimuth or the houre of the day.

First, With the Radius of the line of Chords, upon the center C describe the fundamentall Circle ZH NO representing the Meridian, and draw the
<div align="center">C   diameter</div>

diameter H O for the Horizon, and Z C N at right
angles thereto, Z N being the *Z nith* and *Nadir*
points. Then by your line of Chords set the La-
titude of your place (which let be 51 *gr*. 32′) from
Z to E, and from N to Q, drawing the line E C Q
for the Equinoctiall, and at right angles thereto, the
line M P for the axis of the World, P representing
the North, and M the South Pole

Secondly, Supposing the Sun to have 20 *gr*. of
North Declination, take 10 *gr*. (the Semitangent
of the Declination) out of the line of Tangents, and
set it from C to G. Likewise, take 20 *gr*. (the De-
clination) from your line of Chords, and set that
distance upon the Meridian from E unto D, and
from Q unto K: then describe the arke of a Circle
which shall passe directly through the points D G K,
the center whereof will alwayes fall in the line C P
if it were extended, and this arke D G K shall be
the line of the Suns course when his Declination is
20 *gr*. from the Equinoctiall Northward.

Thirdly, Supposing the Altitude of the Sun to
be 50 *gr*. take 25 *gr*. (the Semitangent of the Alti-
tude) out of the line of Tangents, and set that di-
stance from C to F. Also take 50 *gr*. (the Alti-
tude) from your line of Chords, and set them upon
the Meridian from H unto A, and from O unto B,
drawing the arke A F B, the center whereof will fall
in the line C Z being extended, and this arke shall
represent the Almicanter of 50 *gr*. And where
this parallel of Altitude crosseth the parallel of De-
clination, which is at *S*, that is the place of the Sun
at the time of the Question: Therefore, if you draw
the

the arke of a circle which shall passe through the
points M S P, it shall represent the houre of the day;
and another arke through Z S N shall represent the
Azimuth of the Sun at the same time.   And the
distance C T being measured on the Tangent line
will fall upon 20 *gr.* 4', the double whereof is

40 *gr.* 8′, which is the Azimuth of the Sun from the East or West, and the complement thereof to 90 *gr.* is the Suns Azimuth from the South.
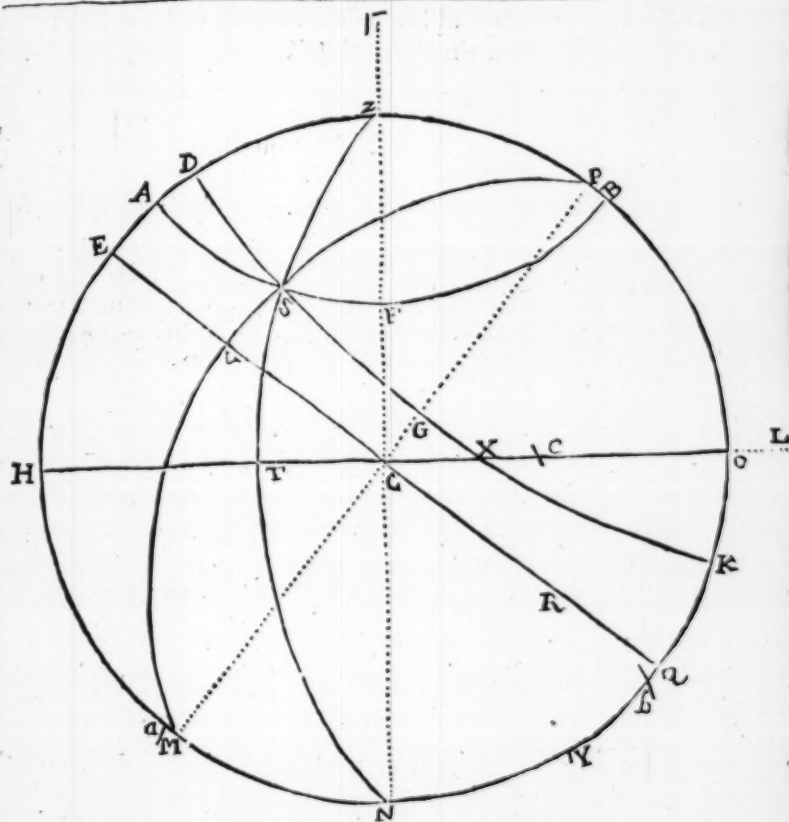
    ❁ 1. The centers of the parallels of declination, and of the parallels of altitude, may readily be fouud by the Scale it selfe; as in this projection, having found the point F upon the line Z C, extend the line Z C without the circle, and becaufe the Suns altitude is 50 *gr.* take therefore out of your Scale the Secant of 40 *gr.* (the complement of the Altitude) and fet that diftance from C to I, fo fhall the point I be the center of the parallel of Altitude.

    Or take the Tangent of 40 *gr.* out of your Scale and place it from F to I, either of which will fall in the point I, the center of the parallel of 50 *gr.*

    In the fame manner may the center of the parallel of Declination be found, by taking out of your Scale the Secant complement of the Declination, and fetting it from C, upon the line C P, (being extended) and where that diftance ends, that is the center of the parallel.

    Or the Tangent complement of the Declination being fet from G on the line G P fhall give the center alfo.

¶ 2. For the finding of the centers of the Hours
and Azimuths, the Scales of Secants and
Tangents will much help you ; So the Azi-
muth from the South being 49 gr. 52', if
you take the Tangent thereof out of the
Scale of Tangents, & set it upon the Horizon
                                    from

from C to L: the point L shall be the center of the circle N T Z.

Or the Secant of 49 *gr* 52' being set from T, that also shall give you the center L as before.

The center of the Houre-circle is found in the same manner, for the houre from the Meridian being 31 *gr*. 28', if you take the Tangent thereof out of your Scale, it shall reach from C to R , the point R being the center of the Houre-circle M V S P.

Or the Secant of 31 *gr*. 28', being set from V, shall give the point R for the center of the same houre.   And in this manner may any Houre or Azimuth whatsoever be drawn.

Many other propositions in Astronomie, may be wrought upon this projection, and indeed any of the 28 cases of Sphericall Trigonometry , may by this kinde of projection be easily illustrated and resolved, which will cleerely informe the fancie in the resolving of Sphericall Triangles. An Example or two for practise shall be,

1. *To finde the Suns Amplitude.*

In this projection, the Amplitude from the East or West is represented by the line C X, take therefore the distance C X in your compasses, and apply it to the line of Tangents , (counting every degree

of

of the Tangents to be two degrees) and where it resteth, that shall be the Amplitude from the East or West, which will be found to be 33 *gr.* 22′.

Or if you lay a Ruler upon Z and X, it will cut the Circle in Y, and the distance N Y being measured on the line of Chords, shall give the Amplitude also.

2. *To finde the distance of the Sun from the Zenith.*

The distance of the Sun from the Zenith is the arke Z S, therefore to finde the quantity thereof, you must first finde the pole of the circle N S Z, which is done after this manner.

Lay a Ruler from Z to T, and it will cut the circle in *a*, then take in your compasses a quadrant of the outward circle, and set it from *a* to *b*, then lay a ruler from Z to *b*, and it shall cut the Horizon in *e*, which point *e* is the pole of the circle Z T N.

Now to measure the arke Z S, you must lay a ruler upon *e* and S; which will cut the outward circle in the point A, so shall A Z, being measured upon the line of Chords, give you the quantity of degrees contained in the arke Z S, which will be 40, equall to the complement of the Suns Altitude.

     ⟨ This latter proposition was inserted rather to shew how the arke of any great Circle of the Sphere (the sides of all Sphericall Triangles being such) may be measured, then for any need there was to finde the distance of the Sun from the Zenith, for that mighe have
been

been more eafily effected, it being only the
complement of the Suns Altitude ; but ac-
cording to this operation, may the fide of
any Sphericall Triangle whatfoever be mea-
fured.

The line of Sines alfo will project the *Analemma,*
as Mafter *Gunter* fheweth, if this propofition be
added.

How to divide any line given, into fuch parts as
the Scale of Sines is divided.

Which propofition may be done by that which
is fet down in the 1 *Chapt.* For if A D *m n* C were
parts or divifions made equall to thofe upon the
Scale of Sines, and C E were a line in the fame
manner to be divided: After you have prepared
your work as is there prefcribed, you need only to
take the leaft diftances between the points C *n m* D
and the line A E, and infert the fame into your given
line, fo fhall the divifions thereof be proportionall
to the line of Sines.
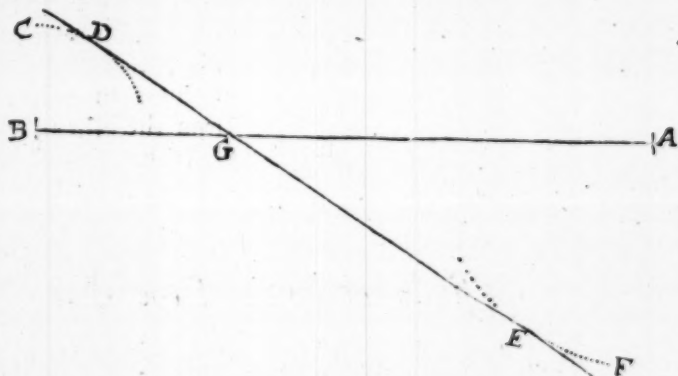
CHAP. VII.

Of the line of Verfed-Sines.

THe generall ufe of this Scale is principally to
refolve thefe two Sphericall Cafes.   Firft,
By having three fides of a Sphericall Trian-
gle, to finde an angle.   Secondly, By having two
fides

fides and the angle comprehended, to finde the third fide. According to which two generall cafes you fhall finde particular examples; namely, the firft and third *Sections* of this *Chapter* futable to the firft Cafe : and the 5 *Section* anfwerable to the fecond.

<div align="center">

S ᴇ ᴄ ᴛ. 1.

To finde the Suns Azimuth.

</div>

Irft, finde the fumme and difference of the com-
plement of your Latitude, and complement of
the Suns altitude. Then having made A B equall
to the length of the whole Scale, count upon the
fame Scale the fumme and difference before found.

After this, take with your Compaffes the diftance from the Suns place to the fumme, and fetting one foot of that extent upon B, with the other defcribe the arke C D. So again, take the diftance upon the

<div align="center">

E   Scale

</div>

Scale from the Suns place to the difference, and with
that extent upon the center A, deſcribe the arke
E F: Which done, draw the ſtreight line D E, ſo
as it may juſtly touch thoſe two arks, cutting the line
A B in G: ſo ſhall B G (being meaſured upon the
Scale, from the beginning of it) ſhew the Azimuth
from the South. And A G meaſured upon the ſame
Scale will give the Azimuth from the North.

<div align="center">

S ᴇ ᴄ ᴛ. 2.

</div>

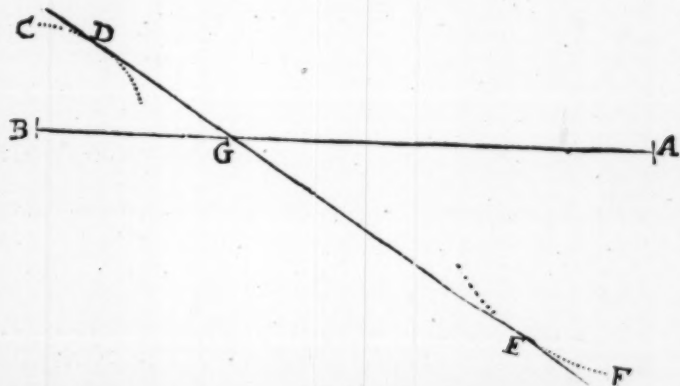## To finde the Amplitude of the Suns Riſing or Setting.

IF you ſuppoſe the Sun to be in the Horizon, or
oo *gr.* high, and ſo the complement of the Alti-
tude to be 90, and if (upon theſe ſuppoſitions) you
work as in the laſt *Section* is ſhewed , then ſhall B G
give the graduall diſtance of the Suns riſing or ſet-
ting from the South; A G from the North, and
from the midſt of the line to G, is the Amplitude
from Eaſt or Weſt.

<div align="center">

S ᴇ ᴄ ᴛ. 3.

</div>

To finde the houre of the Day.

MAke A B equall to the whole Scale, as before:
and count from the beginning of the Scale to
the Suns place what number of degrees there are;
the ſame number ſhews the graduall diſtance of the
Sun from the North Pole. Of this diſtance and
the complement of your Latitude, finde the ſum and
<div align="right">dif-</div>

difference, and count them both upon the Scale, as
was done before. Then again, count thereon alſo
the complement of the Suns altitude : Upon which
point, ſetting one foot of your Compaſſes, extend
the other to the forenamed ſumme; and with that

extent upon the center B deſcribe the arke C D.
Again, ſetting one foot of your Compaſſes upon
the complement of the Suns altitude, extend the o-
ther to the forenamed difference, and with that ex-
tent upon the center A, deſcribe the ark E F. Laſtly,
draw the ſtreight line DE, which only touching the
two former arks, may cut the line A B in G: ſo ſhall
A G (meaſured on the Scale, from the beginning
of it) give the degrees of the Suns diſtance from the
South. Theſe may be turned into houres, counting
15 *gr.* for one houre, and 1 *gr.* for 4 minutes of an
houre.

E 2   SECT.

Sect. 4.
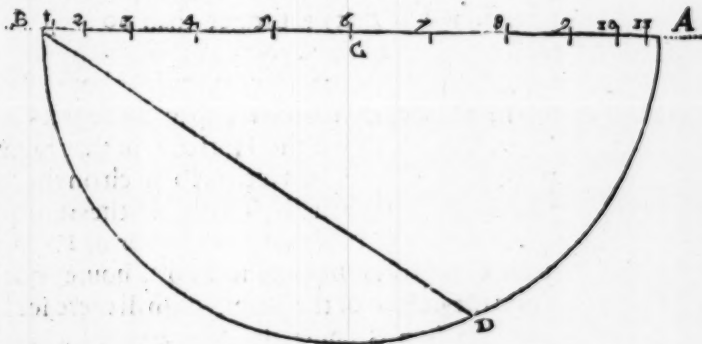
To finde the Semidiurnall and Seminocturnall arks.

IF you suppose the Suns altitude to be co gr. and so the complement of it to be 90, and then work as is directed in the 3. *Sect*. of this *Chap*. then shall A G give the Semidiurnall arke, and B G the Seminocturnall arke: Each of these turned into Houres and minutes, and doubled, will give the length of the Day and Night.

Sect. 5.

The Suns place being assigned in any point of the Eclipticke, to finde his Altitude at all houres.

BY this, may Tables of the Suns Altitude be made to all houres, the Sun being in any Signe of the Zodiacke, whereby many particular Instruments for finding the houre of the day, may be made, as Rings, Quadrants, Cylinders, and such like.

Draw the line A B, and upon it, with C A or C B, equall to halfe your Scale, describe a Semicircle. Then count upon your Scale the Suns distance from the North Pole, as was done in the 3. *Sect*. of this *Chapt*. and in the same manner also finde the summe and difference of this distance and the complement

plememt of your Latitude.   Then take the diſtance
of the ſaid ſum and difference in your Compaſſes,
and ſet it upon the Semicircle from A to D, and
draw B D. Now becauſe A B is equall to your
whole Scale, you may divide the ſame into houres,
by transferring each 15'*gr.* from your Scale to the
line.   This done, take the leaſt diſtance from the
point 1 to the line B D, and ſet one foot of this di-
ſtance upon the forenamed difference counted upon
the Scale, and let the other foot ſtand further on-
wards upon the Scale, and where it falls, it ſheweth
how many degrees that houre of 1 is diſtant from
the Zenith.   Or if you count the degrees from the
middle of the Scale, it ſhews the Altitude of the
Sun in that Houre. Thus doe for the points of
2, 3, 4, &c. and you ſhall in the ſame manner
finde their Altitude : And if you go on to the end,
you ſhall (moſt commonly) finde your Compaſſes
at laſt to reach beyond the middle of the Scale.
                                                [This

[This alwayes, and then only, happens, when the
fum (found at first) is greater than 90 *gr*.] Look
then how much it is beyond, for so many degrees
is the Sun below the Horizon at that houre of the
night: Or (which is all one) so many degrees is the
Sun elevated above the Horizon in that Signe or
point of the Eclipticke which is so much on the other
part of the Equinoctiall. That is, If the Suns place
given were the beginning of *Taurus* or *Virgo*, and
your Compasses (suppose at the 9th. houre) goe be-
yond the 90th *gr*. of the Scale, you shall there see how
low the Sun is under the Horizon at 9 a clock at
night, or at 3 in the morning. And the same also
sheweth how high the Sun is at 9 in the morning,
or at 3 afternoon, if his place were in the entrance of
Scorpio or *Pisces*, which two Signes are so much be-
yond the Equinoctiall on the other part, as *Taurus*
and *Virgo* are on this side.

S ᴇ ᴄ ᴛ. *6.*

All Proportions in Sines alone, where the Radius stands first, may be wrought upon this Scale, without any protraction at all.

THe manner of the work will best appear by an
Example. Let the proportion set down before
in Sines alone be here repeated. The terms stand
thus :

> As the Radius, to the Sine of 23$\frac{1}{2}$;
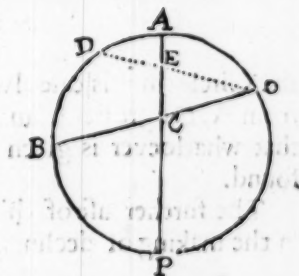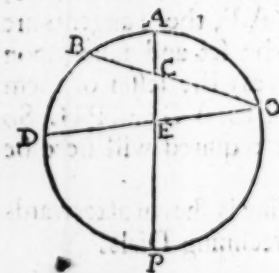> So the Sine of 10, to the Sine of what?

Take

Take the sum and difference of the second and third
arks, the sum is 33½, the difference is 13⅕: count
these both upon the Scale, and there take their di-
stance: apply the same to the middle of the Scale; so
as that the same number of degrees may be above
90, that is below ; so shall the degrees either above
or below, be about 4; and this is the Sine required
for a fourth proportionall to the former.

CHAP. VIII.

How to work proportions in Sines and Tangents, by the lines of Versed-Sines and ⊙.

DEscribe a Circle, as A B D, of the same Ra-
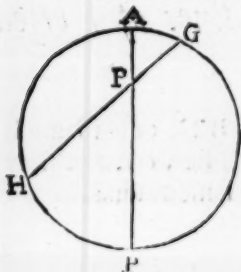dius with the line ⊙. The Versed Scale is in
length four times the same Radius.

Let the Sines (given or required) be measured out
of ⊙, and let them be set upon the Radius from A,
to A C or A E.

Let

Let the Tangents (given or required) be meaſured out of the Verſed Scale, from 90 to 00, or to 180, which are 90 Chords belonging to 90 equall parts of the Semicircle A B D P, and the ſame Tangents muſt ever be ſet upon the Circle from A, as A B, A D:

Then draw a right line through the firſt and third of the given terms, as from B and C to O; and another right line from O to D or to E.

So the fourth terme required ſhall be either the Sine A E, or the Tangent A D, each to be meaſured in its proper Scale.

If the Radius be ingredient in the proportion, then this of Sines and Tangents may be wrought by the draught of one line: ſee this third Scheme. The Sine is to be taken or ſet on the Radius from A, as A F, the Tangents are to be ſet and taken upon the Circle (in this caſe alwayes) the leſſer of them from A, the greater from P; as A G and P H. So that whatſoever is given or required will here be found.

The further uſe of this line is ſhewn afterwards in the making of declining reclining Dials.

CHAP.

CHAP. IX.

To finde the declination of a Plain.

TO effect this, there are required two obser-
vations: the firſt is of the Horizontall di-
ſtance of the Sun from the pole of the plain,
the ſecond is of the Suns Altitude, thereby to get
the Azimuth. And theſe two obſervations muſt be
made at one inſtant of time, as neer as may be, that
the parts of the work may agree together the better.

1 For the horizontall diſtance of the Sun from
the pole of the plain: Apply one edge of a Quadrant
to the plain, ſo that the other may be perpendicular
to it, and the limbe may be towards the Sun, and
hold the whole Quadrant horizontall as neer as you
can conjecture: Then holding a threed and plum-
met at full liberty, ſo that the ſhadow of the threed
may paſſe through the center and limbe of the Qua-
drant, obſerve then the degrees cut off by the ſha-
dow of the threed, and number them from that ſide
of the Quadrant that ſtandeth ſquare to the plain,
for thoſe degrees are the diſtance required.

2 At the ſame inſtant obſerve the Altitude of
the Sun, theſe two will help you to the plains de-
clination by the rules following.

Firſt, By having the Altitude, you may finde
the Azimuth by the 1. *Sect.* of the 8. *Chap.* then by
comparing the Azimuth and diſtance together, you
may finde the plains declination in this manner.

When you make your obſervation of the Suns

horizontall diftance, marke whether the fhadow of
the threed fall between the South and that fide of
the Quadrant which is perpendicular to the plain.
For,

1. If the fhadow fall between them, then the di-
ftance and Azimuth added together, do make the
declination of the plain , and in this cafe, the decli-
nation is upon the fame coaft whereon the Suns A-
zimuth is.

2. If the fhadow fall not between them, then
the difference of the diftance and Azimuth is the de-
clination of the plain , and if the Azimuth be the
greater of the two , then the plain declineth to the
fame Coaft whereon the Azimuth is: Otherwife,
if the diftance be the greater then the plain declineth
to the contrary Coaft to that whereon the Suns A-
zimuth is.

℘ Note here further, that the Declination fo
found is alwayes accounted from the South,
and that all Declinations are numbred from
either South or North towards either Eaft
or Weft, and muft not exceed 90 *gr.*

1. If therefore the number of declination ex-
ceed 90, you muft take the refidue of that number to
180 *gr.* and the fame fhall be the declination of the
plain from the North.

2. If the number of declination doe exceed
180 *gr.* then the exceffe above 180 gives the plains
declination from the North, towards that Coaft
which is contrary to the Coaft whereon the Sun is.

℘ And

⸿ And here note, that wheresoever in this
Chapter the use of a Quadrant is required,
the Scale of Chords will effect the same; if
upon a piece of plain board you describe a
Quadrant, whose sides may be parallel to
the edges of the board, upon which you
may set off the Horizontall distance and
Altitude, which will performe the work
thereof when a Quadrant is not at hand.

CHAP. X.

OF DIALS.

To draw upright declining Dials, by the lines of Sines and Tangents.

THe declination of the plain being found by
the last *Chap.* Upon your plain describe a
rectangled parallelogram, in which let the
sides A B and C D be perpendicular to the Hori-
zon, and each of them equall to the Tangent of
your Latitude: and let A C and B D be equall each
of them to the co-tangent of your Latitude, and let
B D be prolonged if need be.

Then taking that side of the parallelogram (for the
houre of 12) which looketh towards that coast unto
which the plain declineth, as here namely, the side
A B; and on that line having assumed the superiour

angle A in South-declining-plains, or the inferiour
in North-decliners, for the center of your Diall:
Let B E and C G be made equall to the Sine of the
plains declination, so A E being drawn, shall be the
substilar, and A G shall be the houre of 6. Then
from E, raise E F perpendicular to A E, and make
A 12 and E F equall to the co-fine of the declinati-
on: and if you draw A F, the same shall represent
the Axis, and the angle F A E sheweth how much
the same is to be elevated above the substylar.   A-
gain, make A H equall to the co-fine of your Lati-
tude, and draw H 6 parallel to A B; which will cut
A G in the point noted with 6. To this A 6, let
A 6 also beyond the center be made equall, and then
draw the lines 12 6 and 12 6, which lines must
have the houre points set upon them; and to per-
forme that worke doe thus.

Draw upon paper, or some other plaine, the line
L M, upon which set L R and R M, each of them
equall to your Tangent of 45 *gr.*   Then make R N
equall to the Tangent of 30, and R O equall to the
Tangent of 15, so shall you have points to finde all
the houres, and if you desire halves and quarters, you
must also put their Tangents into the same line R M.
Being thus prepared, if you would divide the lesser
line 12 6 into its requisite parts, take the same
line in your Compasses, and with it, upon L as a
center describe the arke P Q, and from M draw
M P, which may only touch the same arke.   Then
from N take the least distance to the line M P, and
the same will reach from 12 to 11, and from 6 to 7;
so the least distance from O to the line M P, will

<div align="right">give</div>

give from 12 to 10, and from 6 to 8. And the least distance from R will reach from 12 or 6 to 9.

In the same manner you must divide the larger line 12 6.   Take it out of your Diall, and with it describe the arke S T from the same center L, and draw M S touching only the same arke. Then the least distances from N, O, R, to the line M S, will give the points or distances 12 1; 6 5; and 12 2; 6 4; and 12 3; or 6 3.   These upon the South-decliner; the like may be done upon the North-decliner. Lastly, from the center A, through these points you must draw the Houre-lines.

An

An upright plain declining from
the South towards the East, 30 gr.

An upright plain declining from
the North towards the East, 30 gr.

In both these Plains.

A B =tang. Lat. ⎫ a rectangle
                ⎬ parallelog.
BD=co-tang.lat. ⎭
B E=C G=sine declination.
A 12=E F=co-sine declina.

A H =co-sine I at.
A 6=A 6.
R L=R M=tang.45.
R N=tang. 30.
R O=tang.15.

CHAP

CHAP. XI

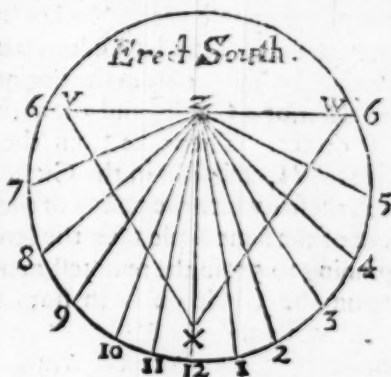## Of the Horizontall and full South Dials.

THese are done more eafily, for having made ZX for the line of 12, and W V perpendicular thereto for the two fixes, in them both make Z X equall to the Radius, and in the Horizontall let ZV and Z W be equall (each of them) to

the Sine of your Latitude : in the South plain let Z V and Z W be (each of them) equall to the cofine of your Latitude. Then draw the lines X V and X W, and divide them as was now fhewed in the declining plains ; fo may you from the center Z, and

and thefe points, draw all the houres, as you fee in thefe figures.

The Styles are to ftand over the line of 12 : that in the Horizontall muft be elevated fo much as your Latitude comes to; the other according to the complement of your Latitude.

The upright North plain is the fame with the South, only turned upfide down, and the courfe of the figures altered.

The Eaft and Weft upright plains may be made by the Tangent line, in fuch manner as others have prefcribed.

CHAP. XII.

Of the Scale of Horizontall Spaces.

FOr the Horizontall plains in fpeciall, there is a peculiar Scale by which the houres may fodainly be defcribed, to any Latitude between 30 and 60 degrees.

The manner of which work is eafie. For you

G　　　　　　have

have the numbers from 30 to 60 five times repeated, ſerving for the five houres in ſo many Latitudes.　Suppoſe then a Horizontall Diall were to be deſcribed for the Latitude of 51½ *gr.* Firſt, by the Radius (which is from the beginning of the line to R) deſcribe a Circle, and draw the line of 12 from the center.　Then take from the beginning of the line to VI, and ſet it in the Circle both wayes from 12, theſe two are the points of the two ſixes. Again out of the ſame Scale take the length from the beginning to 51½ in the remoteſt numbers, and ſet that upon the Circle on both ſides 12, theſe are the points of 5 and 7.　So from the beginning of the Scale to 51½ in the next remoteſt numbers, being ſet as the other were, will give the points of 4 and 8. The third 51½ will give the points of 3 and 9. And the fourth gives 2 and 10.　The laſt gives 1 and 11.

❧　The Chord line that is fitted to this Horizontall Scale, is of good uſe in other delineations: But the further uſe of theſe two joyntly, muſt be referred to another place.
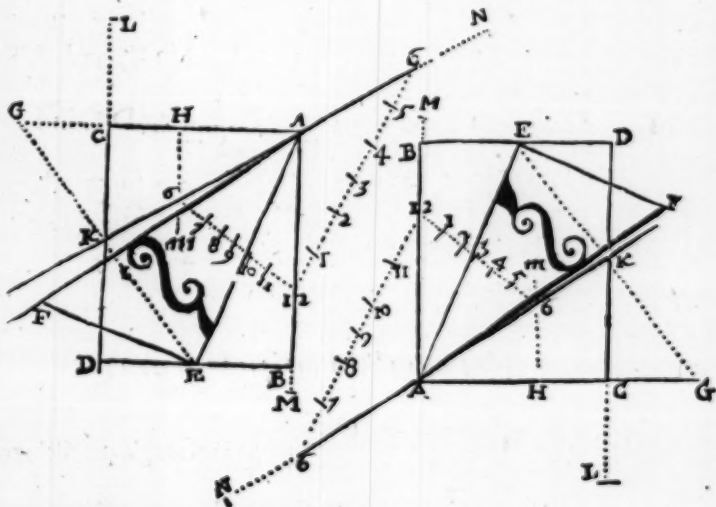
CHAP. XIII.

How to draw upright declining Dials when the Latitude of the place is very little or very great.

IN the work of the 10 *Chap.* it may fall out that either the Tangent or co-tangent of the Latitude may be too great, ſuch as the Scale wil not afford.
This

This will frequently fall out in the new Latitude of
re-in-cliners: to remedy that inconvenience, I have
added these helps.

Where the Latitude is but small

1. A B=C D=tang. of Latit. ⎱
2. B D=A C=Radius.   ⎰ A rectangle parallelo.
3. B E=C G=Sine declination.
4. A 12=E F=cosine of declination.
5. A H=cosine of Latitude.
6. H m=A B.
7. Draw E G, it will cut C D in K. A K is the line
   of fix: it cuts H m at 6, make A 6=A 6, on both
   sides, and draw 12 6; 12 6; and divide them
   as the other are in the 10 *Chap.*

Or

Or you may draw B C the Diagonall, and E K
≈ thereto, and so omit C G.

Or you may make the ∧ D E K=to your Latitude,
and so omit the two former.

Or thus.

After the 1, 2, 3, 4, 5, you may omit the 6. Then
7^{thly}. Draw E G it will cut C D in K, and A K
is the line of six.

Then lay a Ruler from 12 to H, cutting D C in L.

Make 12 M=C L, and A N=A K.

So shall K M, M N, be ≈ to the two former lines
6 12; 6 12; and may supply their Offices som-
what better, because they are larger.

Where the Latitude is great.

1. A B=C D=Radius,   ⎫ a rectangle parallelogr.
2. B D=A C=co-tang. lat.   ⎭

3. B R=C G=Sine declination.
    G R a right line cutting D B in E.
    A E Substilar.
    A G houre of 6.

4. G P=R T=A 12=co-sine of declination.
    T P a right line, cutting B D in O.
    A E F a right angle.

5. E F=E O.
    A F the Style.

6. A H=co-sine Latitude.

H *m* ≈ A B, cuts A G in 6.
A 6 = A 6, on both sides.
Draw 6 12; 6 12, &c.

Or after the 1, 2, 3, 4, 5.
6. Draw 12 H, it cuts D C in L.
    Make 12 M = L C: and A N = A G.
        Then G M, M N shall be ≈ to 6 12; 6 12:
            and may therefore supply their uses.

CHAP. .

CHAP. XIIII.

Concerning Reclining and Inclining Plains, how to draw houres upon them.

THey may be referred to a new Latitude, in which they shall stand as upright plaines: and then the delineation will be the same with those in the 10 *Chap.*
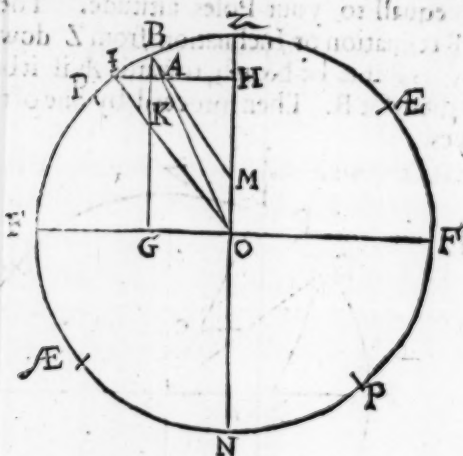
The Meridian line is not here to be taken for the line of 12 at mid-day (for it often reprefents the mid-night) but for that part which helps to deferibe the Diall.

1. The firft thing to be done upon thefe plains, is (by fome levell) to draw the Horizontall, and then the Verticall line perpendicular thereto.

2. Next is the placing of the Meridian upon the plain, in a true pofition.   In direct plains that $_{in}^{re}$-cline, and in upright decliners, the Meridian is the fame with the plains Verticallline.   In Eaft and Weft $_{in}^{re}$-cliners, it is the fame with the horizontall line.   In the reft, it afcendeth or defcendeth from the horizontall line, and muft be placed according to the rules hereafter given.

3. For which purpofe, In the Circle FZFN(made of the fame Radius with that of Sines and Tangents, &c. upon the Ruler) fet F P, Z Æ, F P, N Æ,

each

each equall to your Poles altitude. Then count
the Reclination or Inclination from Z downwards,
towards P if it be North, towards Æ if it be South;
and there set B. Then proceed by one of these two
wayes.

I.

$ZB\genfrac{}{}{0pt}{}{re}{in}$-clination.

B G $=$ Z O

O C co-sine of declination.

R C $=$ F O.

O R D Radius.

D S $=$ B O.

Out of this Structure will follow.

D Æ new Latitude lesse than 90.

O R co-sine of new Declination.

D S Sine of Meridians ascension or descension.

II.

II.

Z B$^{re}_{in}$-clination.

B G ≈ Z O.
O A Sine of declination.
H A I ≈ F O.
I O Radius.
A M ≈ I O.

Out of this Structure will follow.

O K is the sine of Z D or N D in the former figure,
   where the new Latitude D Æ will be found.
H O Sine of new declination.
A M co-sine of Meridians ascension or descension.

℃ How all $^{re}_{in}$-clining plains (being counted as
   upright in their new Latitude) are to be taken;
   whether as North or South decliners.

As also,

¶ How the Meridian line is to be placed, whe-
ther afcending above, or defcending below
the horizontall line: and from which end
of that line, whether that which looks the
fame way with the declination of the plain,
or that which looks the contrary way.

In North
re-cliners.
in

If D fall
below P,
the

  Recliners are North
plains, and the Meridian
afcends above the horizon-
tall line, from that end of
it which looks to the fame
Coaft of declination.

  Incliners are South plains,
and the Meridian defcends
below that end of the hori-
zontall line, which looks
to the contrary Coaft of
declination.

If D fall
above P,
the

  Recliners are South
plains, and the Meridian
goes below: contrary,

  Incliners are North plains,
and the Meridian goes a-
bove the end looking the
fame way with declination.

H

In

In South re in·cliners

If D fall above Æ, the

Recliners are North plains, and the Meridian goes above the horizontall line, from the fame end with the Coaft of declination.

Incliners are South plains, and the Meridian goes below the horizontall line, from that end which is contrary to the Coaft of declination.

If D fall below Æ, the

Recliners are North plains, and the Meridian goes below the horizontall line: contrary,

Incliners are South plains, and the Meridian goes above the horizontall line, from that end which looks to the Coaft of declination.

If

Same end in Recliners.

If D fall into P, both re-clincrs, are called Po-in lar plains, and the Meridian, in both, afcends from the

contrary end in Incliners.

Recliners are North plains, and the Meridian afcends from the fame; defcends from the contrary end to that which looks upon the Coaft of declination.

If D fall into E, the

Incliners are South plains, and the Meridian afcends from the contrary; defcends from the fame end that looks upon the Coaft of declination.

Recliners are North plains, declining from North,

So much as the complement of their re-clination comes to. in

☾ Eaft & Weft

This is their new declination, & their new Latitude is the complement of the Latitude of your place.

Incliners are South plains, declining from South,

If

H 2   4 For

4. For that which follows, take notice of these four things. First, That from D to the neerest Æ (measured by the line of Chords) gives the new Latitude, in which the $\frac{re}{in}$-clining plain, is an upright declining plain. Secondly, That O R (measured upon the line of Sines) gives the complement of the plains new declination in that new Latitude: this New declination is to the same Coast with the Old, but alwayes lesse in quantity than it. Thirdly, That D S (measured upon the Sines) gives the quantity of the Meridians ascension or descension. This gives the quantity, the former rules gave the Coast. Fourthly, That in the description of the Diall, you must only make use of the new Latitude, and new Declination: having nothing to doe with the other.

5. Having the former things known, you must (by the Tangent and co-tangent of the new Latitude) describe your Rectangled Parallelogram (as in the 10 *Chap.*) and according as the plain was discovered to be a decliner from the North or South, you must make choice of your center, place the substylar, style, and fix a clock line, by help of the Sine and co-sine of the new declination, and new Latitude, and then prick down and draw the houres, all in the same form that was before shewed in the 10 *Chap.* for upright decliners.

This for the Dials description.

6. Lastly, for placing your Diall. First, Consider which way, and how much, your Meridian

afcended or defcended from the horizontall line.
Then goe to your plain, and there draw the fame
Meridian line anfwerably, fetting off fo many de-
grees by your Scale of Chords. When this is
done, take your paper defcription, and lay the Me-
ridian of it, either upon, or elfe parallel to, the Me-
ridian drawn upon the plain, and take care to place
it the right way, namely fo, as that the imaginary
ftyle of your paper (or a reall pattern of the ftyle
cut fit and fet upon the paper Diall) may point into
the North or South Pole, according as the plain is
efteemed to be a North or South plain. After this
is performed, you may transfer each houre from
the paper to the plain, and fo finifh all the work.

CHAP. XV.

Concerning full East and West re-in-clining plains.

HEre in this fort of plain, you are only to take
notice, that the new Latitude (wherein they
ftand as erect plains) is ever the complement
of your own Latitude. And the new declination
(in that Latitude) is the complement of their
$^{re}_{in}$-clination. By knowing thefe, you may defcribe
the diall according to the 10 *Chap.* The Meridi-
an line (in all thefe) lyeth in (or parallel to) the ho-
rizontall line. All which things will appear alfo
out.

out of the former figures, if according to them you
ſhould make a draught , and ſuppoſe your plain to
decline 90 degrees, as all theſe Eaſt and Weſt plains
do.　All other things will follow of themſelves,
agreeable to other plains.

CHAP. XVI.

Concerning re-in-cliners, that are direct, or have no declination.

IF the line C B be plaɔed (as is preſcribed in the
former figure) and drawn quite through, it will
repreſent your plain that is $_{in}^{re}$-clining　towards
the North, and without any declination.　So alſo
B L, if it be drawn quite through , will repreſent
ſuch plains as $_{in}^{re}$-cline towards the South, and have
no declination.

For which lines ſo drawn (or imagined only) you
may gather (according to the former rules) which
of the Poles (A or X) is elevated , and how much it
is elevated (which is ſhewed by the arke C A or
L X.)　You may alſo ſee which end of the Meridi-
an is to be taken for the ſubſtilar line, over which
(in theſe direct plains) the ſtile is ever to be erected,
and muſt ſtand.

Then for drawing the houres , you have no more
to doe, but to deſcribe an Horizontall Diall to that
elevation,

elevation, which is due to the plain. The manner whereof is shewed before in the 11 *Chap.*

CHAP. XVII.
How to deal with those plains, where the Pole is but of small elevation.

SUch plains whose styles lie low, cannot have the houre-lines distinctly severed, unlesse the center of the Diall be cast out of the plain. In such cases therefore the Diall is to be made without a center, in this manner.

1. Place A B the Meridian, A E the substilar, A F the style, by the rules before given in the 10 and 13 *Chapt.* omitting what is done for the line of six, being here of no consequence.

2. Finde the plains difference of Longitude by the 18 *Chap.* following.

3. Assume any two points in the substilar A E, as at R and S, and through them draw two infinite right lines, at right angles to A E.

4. To the style A F, draw the parallel G H, at any convenient distance, such as you shall think fit, for your new style to stand from your plain.

5. Take the least distance from R to G H, and set it upon the substilar from R to K. So from S to G H, set from S to L.
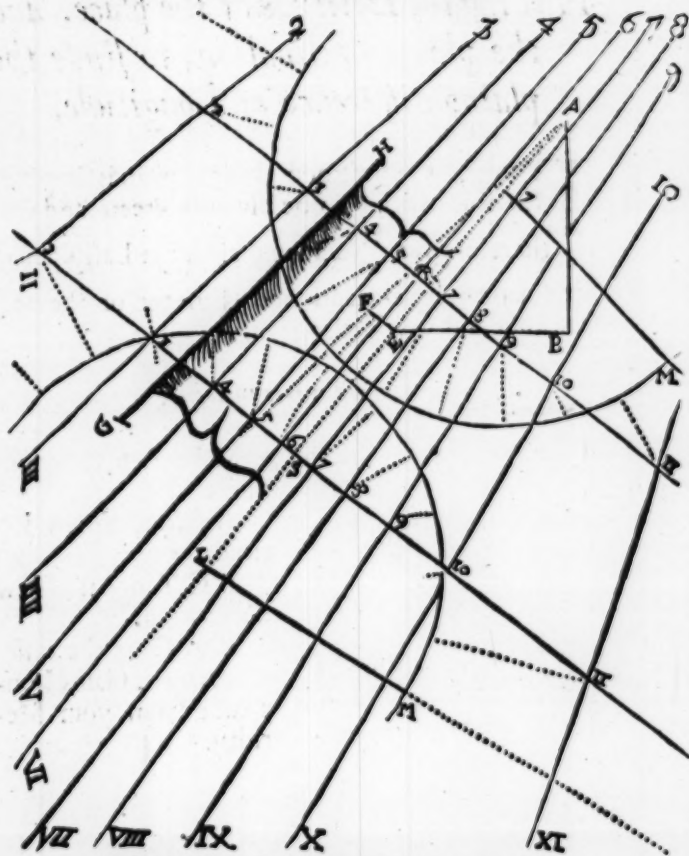
6. Upon

6. Upon the two centers K and L, deſcribe two Circles: And in them both, make the two angles R K M, S L M, equall to the plains difference of Longitude; and ſet it on that ſide the ſubſtilar R S, upon which the Meridian A B ſtandeth.

7. The reſt of the work will be eaſie to finiſh, if you begin (in each circle) from the points at M, to divide them into 24 equall houres; and from the centers to thoſe equall diviſions, draw out lines to cut their reſpective contingent lines in 12, 11, 10, &c. And from each correſpondent houre, you muſt draw the lines 12 12, 11 11, 10 10, &c.

An

An Upright Plaine declining from South towards East 80 gr.

Difference of Longitude 82ᵈ 08′
South Pole elevated   6   12
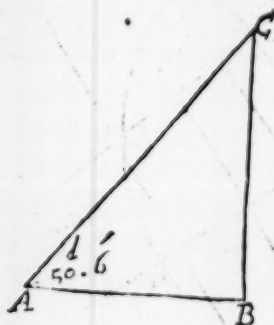   I   CHAP.

CHAP. XVIII.

*Having the Latitude of the place, and
the plains declination, to finde the
plains difference of Longitude.*

IT muſt be underſtood, that the plain is ſuppoſed
(in this work) to be alwayes erect; and that
therefore for $\frac{re}{in}$-clining plains, the Latitude and
declination here mentioned is meant of the new
Latitude and new declination.

Two wayes to doe it.

I.

Make A B C a right
angle.
    A B Sine of new La-
titude.
    B C Tangent of new
declination.
    B A C is the diffe-
rence of the plains Lon-
gitude from your Me-
ridian.

d 6
50 . 6

Make

Make D E F a right angle.

D E Radius.
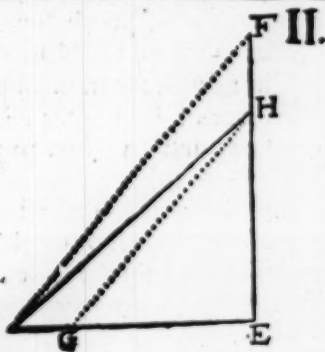
E G Sine of new Latitude.

E F Co-tangent of plains new decli nation.

Draw G H pa- rallel to D F.

H D E is the complement of the difference of Longitude.

Or D H E is the difference it selfe.

If this work be done for upright plains in your own Latitude, which will be needfull in far decli- ners, then instead of the new Latitude and new De- clination here mentioned, you are to use your own Latitude, and the upright plains Declination. The new Latitude and Declination are for re-in-clining plains.

CHAP. XIX.

Of Polar Plains, on which the Pole is not elevated at all.

THose are called Polar plains, upon which neither of the two Poles is elevated at all, but the plaine lies parallel to the Axis, such are the upright East and West: and in every decli-

nation

nation from the South fome one recliner : in every declination from the North fome one Incliner.

The new declination of all Polar plains is their difference of Longitude , in thefe you muſt work by the 10 and 14 *Chap.* to place A B the Meridian, A E the fubſtilar; & for the ſtyle A F, it hath no elevation from the fubſtilar, but is the fame with it. So that the work will be much like that in the 17 *Chap.*

Make G H for the ſtyle , parallel to the fubſtyle A E, at fome convenient diſtance. Then aſſigning any point in the line A E, as S, through it draw an infinite right line perpendicular to A E. And take the leaſt diſtance from S to G H, make S L equall thereto. Upon L defcribe a circle, and make SLM equall to the difference of Longitude , on the fame Coaſt from S L unto which the plain declineth, or to the fame Coaſt upon which the firſt line of 12 namely A B ſtandeth. Then having found the houre points upon the line which paſſeth through S, namely, 6, 7, 8, 9, 10, &c. draw lines through them, all parallel to the fubſtilar A E S L.

CHAP. XX.

Another way to prick down the houre-points , by the Tangent line on the Scale.

L Et the firſt four Sections of the 17 *Chap.* be performed according to the directions there given. After them , you muſt gather the angles

angles at the Pole, by help of the plains difference
of Longitude in this manner. Let the former ex-
ample serve. The difference of that plains Longi-
tude will be 82 *gr.* 08'.

Out of this, take the greatest
number of some just houre;
viz. 75 *gr.* The remainder
is 7 *gr.* 8'. Having then
set down the subftile 00 00,
as in the Margin, write this
7 *gr.* 8'. next under it, to
which adde 15 *gr.* continu-
ally, and you shall produce
all the following numbers
as you there see them. And
note, that in this work 82*gr.*
8'. the difference of Longi-

	gr.	'
3	52	52
4	37	52
5	22	52
6	7	52
Subftyle	00	00
7	7	08
8	22	08
9	37	08
10	52	08
11	67	08
12	82	08

tude will ever ftand againft the houre of 12, if you
work right. Then take the firft number 7 *gr.* 8'.
out of 15 *gr.* the remainder is 7 *gr.* 52'; set this
above the subftyle, and to this number adde conti-
nually 15 *gr.* (or one houre) the numbers will be
produced such as you here see.

When this is done, draw a right line, therein af-
fuming the point S or R. Then upon your Scale of
Tangents, count the numbers 7 08, 22 08, &c.
in the Table, and take them off from the same Scale,
setting them severally from S to *a, b, c, d, e.* So
again, upon the same Scale of Tangents count the
other numbers, 7 52, 22 52, 37 52, &c. and
take them off thence severally, and place them from
S at *f, g, h, i.*

After:

After this, take 45 *gr*. out of your Tangent Scale,
and place it upon this line from S to K both wayes.
Then (as in the 19 *Chap*.) take the leaft diftance
from the point S (in
your Diall) to G H
the fiduciall edge of
your ftyle, and fet-
ting one foot of that
extent upon K, with
the other defcribe
the arke M P, and
from S draw the
lines S P only touch-
ing the faid arks.
Being thus fitted,
you muft from *a*, *b*,
c, *d*, *e*, take the leaft
diftances to the line
L P, and fet them
(refpectively) upon
the contingent line
of your Diall, from
S to 7, 8, 9, 10, 11,
And again, the leaft
diftances from *f*, *g*,
h, *i*, to the line P S,
will give the poins
6, 5, 4, 3, upon the
fame contingent line
of your Diall. Thus
is one of the contin-
gent lines divided
into

into his requisite houres.   The like work must be
done with the other.   For you must take the least
distance from the point R (in your Diall) to G H the
fiduciall edge of your style, and setting one foot of
that extent upon K, with the other describe the arke
T V, and draw the touch-line S V.   Then from
a, b, c, d, e, take the least distance to S V, and set
them on your Diall from R to 7, 8, 9, 10, 11.
So from *f, g, h, i*, take the least distances, and set
them from R to 6, 5, 4, 3, by which means you
have the other contingent line divided into its re-
quisite houres.   The rest of the work for finishing
the Diall will be the same with that in the 17 *Chap.*

But because the tangents upon the Scale goe but
to 63 *gr.* 26', it must therefore here be shewed
how those that exceed that quantity may be sup-
plyed.   Namely thus, Double the number of de-
grees and minutes, and from the sum take 90 *gr.*
so shall the Tangent and Secant of the remaining
arke (both of them put together) give the Tangent
required.   As if in the former example, it were re-
quired to finde the Tangent of 67 *gr.* 8' noted upon
the line by the length S *e*, we must doe thus.   The
double of 67 *gr.* 8' is 134 *gr.* 16', from which take-
ing 90 *gr.* the remainder will be 44 *gr.* 16'.   Accor-
dingly we must first take the Secant of 44 *gr.* 16',
and set it from S to *y*, then take the Tangent of the
same 44 *gr.* 16'. and set it also forward from *y* to *e*,
so shall you have S *e* the whole Tangent of 67 *gr.*
8'. as is required.   Thus doe for any other which
shall goe beyond the Scale.

<div align="right">CHAP.</div>

✿✿✿✿✿✿✿✿✿✿✿✿✿✿✿✿✿✿✿

CHAP. XXI.

A second way for re-in-cliners.

TAke notice of these terms.   1. Verticall distance, is the distance of the plains pole from the Vertex or Zenith of the place.
    2. Polar distance, is the distance of the plains pole from the North pole.

Preparatory works.

The wayes how to effect these are shewed other-where, and are here ta-ken as known.

1. Draw the horizontall line upon the plain, and crosse it at right angles with a Verti-call line.

2. Get the plains $\frac{re}{in}$-clination, and consequent-ly the distance of the plains Pole from the Zenith of the place: which is here called the Verticall distance.

3. Get the plains declination, and alwayes ac-count how much it is from the North.   For that is here called the angle of Declination.
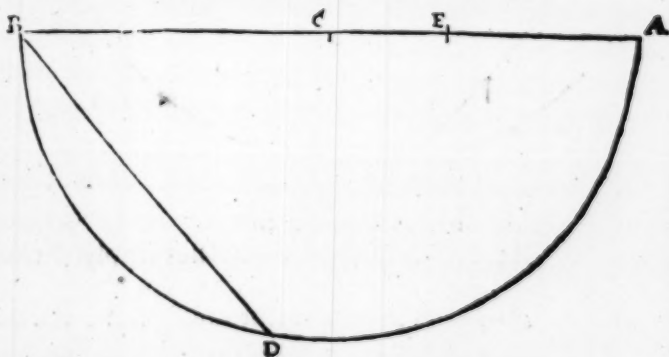
S ᴇ c ᴛ.

<center>S E C T. I.</center>

By the Scale of Versed Sines, how to finde the elevation of the Pole above the plain: and which Pole it is, whether North or South, that is elevated.

First, finde the { The complement of your Latitude,
summe and difference of

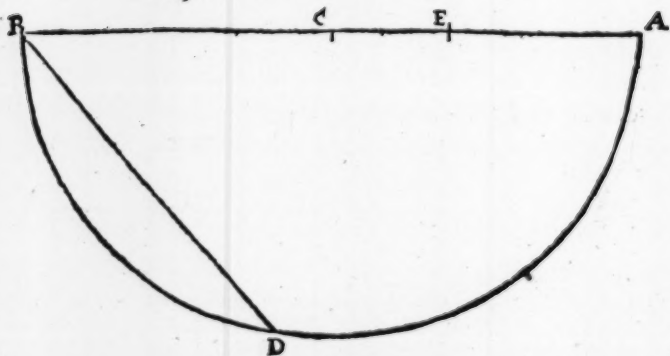{ The plains Verticall distance.

Then take halfe the length of your Versed Scale; and with that Radius (upon the center C) describe the Semicircle A D B. Afterwards, upon the same Scale, count the former summe and difference, and take the length betwixt them, and set it from A

<center>K</center>   to

to D, and draw B D.   Alfo count (upon the fame
Scale) the angle of the plains declination, and fet
that length from B to E, and take the leaft diftance
from E to B D.   This leaft diftance being rightly
applyed to the Scale, namely, from the fore-named
difference forward upon the Scale, will give the di-
ftance of the plains Pole from the North Pole,
which is to be made ufe of hereafter, and is called,
The Polar diftance.

And obferve likewife, that

    If the point of your Compaffes (applyed to the
      Scale) doe fall juft upon 90, then is your
      plain a Meridionall or Polar plain, and hath
      no pole elevated above it.

    If it fall fhort of 90, then is the North Pole ele-
      vated ; and the elevation is fo much as the
      point fals fhort of 90.

    If it fall beyond 90, fo much as it falls beyond,
      fo much is the South Pole elevated.

                            S B C T.

Sect. 2.

To finde the plains difference of Longitude from the South part of your Meridian, and which way the said difference of Longitude is to be taken.

First, finde the sum
and difference of
} {
The complement of your Latitude,
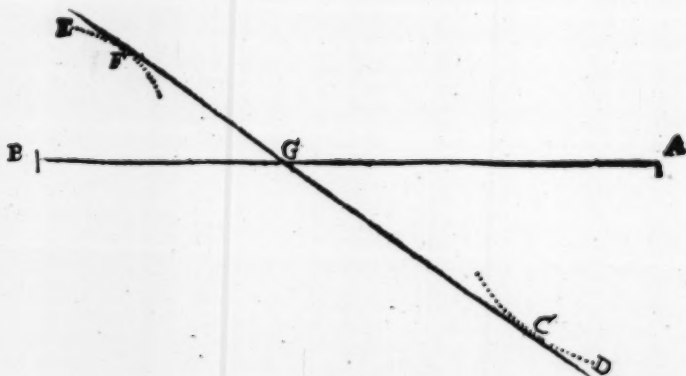
The fore-mentioned polar distance.

Then make A B equall to your whole line of Versed Sines.   And upon your Scale count your difference now found, and the fore-mentioned verticall distance, taking the distance of these two as they are numbered upon the Scale. With which length upon A, describe the arch C D. Take also upon the Scale, from the verticall distance to the fore-mentioned sum, and with that length upon B, describe the arke E F.   Then draw the line F C, so as to touch both these arks, cutting the line A B in G: so shall A G (being measured upon the Scale) give the plains difference of Longitude from the South, which is here required.

℞ This difference of Longitude is to be taken to the same Coast in the heavens unto which the plain declineth, and may afterwards, in the description of the Diall, be easily accounted either from the South or North part of the Meridian, *viz.* so as that the said difference may be alwayes lesse than 90 *gr.*

S E C T. 3.

To finde how much the Subſtilar (or plains proper Meridian) muſt lie from the Verticall line of the plain, and which way.

First, Finde the The Polar diſtance,
ſumme and dif-
ference of   The Verticall diſtance.

Then make A B equall to your whole Verſed Scale. And on the ſame Scale, take the extent from the complement of your Latitude to the difference now before found, with which length, upon A as the center, deſcribe the arke C D. Alſo upon the Scale, take from the complement of your Latitude

to

to the fumme here before found, and with that
length, upon the center B, defcribe the arke E F.
then draw the line F C, juftly touching both thefe
arks, and cutting the line A B in G, fo fhall A G
(being applyed to the Scale) give the quantity of
the angle here required.   According to this angle
the fubftylar line muft alwayes ftand off from the
verticall line of the plain.

Which way muft the Subftilar lie from the Verticall line.

If the plain hath the North Pole elevated upon
  it, then muft the fubftilar alwayes lie from the
  upper end of the Verticall line towards the
  North Pole, fo much as the angle was (in the
  laft *Section*) found to be.

If the South Pole be elevated, then the fubftilar
  lyeth alwayes from the lower end of the ver-
  ticall line towards the fame South Pole, accord-
  ing to the forenamed angle.

If the plain be Meridionall (upon which neither
  of the Poles is elevated) then the fubftilar muft
  doe either, or both; thefe two : according to
  the angle before found.

According to thefe Rules you may place the fub-
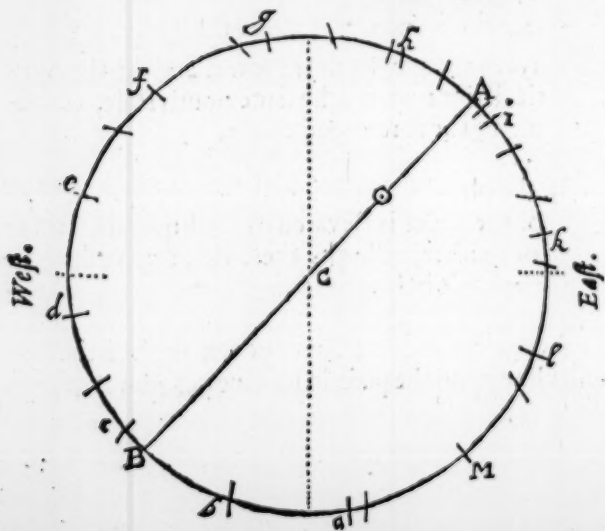ftilar line upon the plain in its true pofition requifite.

S E C T.
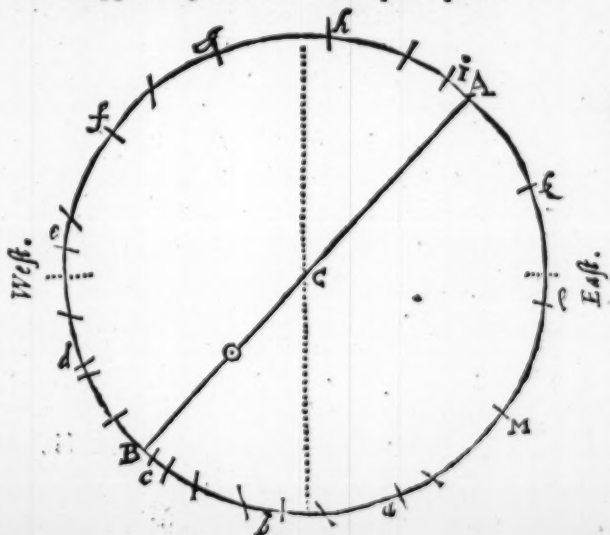
Sɛᴄᴛ. 4.
To draw the houre-lines upon the plain.

Irſt, conſider by the firſt of theſe Sections,
whether it is the North or South pole that is
elevated upon your plain. If it be the North
pole, then muſt the center of your Diall ſtand down-
ward, and the ſtyle muſt point upward to the ſaid
North pole.   But if the South pole be elevated,
then the center of the Diall is to be ſet upward, and
the ſtyle comming from thence muſt point down-
wards into the South Pole.

This being remembred, you may upon paper draw
the right line A C B, and upon C as the center,
with the extent of the line ☉ taken from your Scale,
deſcribe a circle.   If then the $\begin{cases} \text{North pole} \\ \text{South pole} \end{cases}$ be ele-

vated, let the center of your Diall be $\begin{cases}\text{below at B,}\\\text{above at A,}\end{cases}$ and the poles elevation(taken from the Scale ☉)must be set betwixt $\begin{matrix}\text{A and}\\\text{B and}\end{matrix}$ C, namely, at the mark ☉.

Then (counting halfe your Versed Scale as a line of 90 Chords, beginning at 90, and ending at 00, or 180) from that end of the diameter A or B which is the center of your Diall, set off your difference of Longitude (taken out of the Versed Scale as it was now taken for Chords) that way to which the plain declineth, set it off at M, this point M is the point of 12. Then from M, divide your circle into 12 e-quall parts, at *a, b, c, d, e, f, g, h, i, k, l*, and from each of those points through the point ☉, make touches of right lines, cutting again the same circle on the oppofite part into 12 unequall parts.

Laſtly, From the center of the Diall A or B, and through the ſaid unequall parts, draw right lines. Theſe laſt lines ſhall give you 12 of your houres required: And if you draw each of them quite through the center, you ſhall have the whole number of 24, of which, you may take ſuch as are ſutable and neceſſary for your plain.

When your paper Diall is thus finiſhed, you may transfer it to your plain, by laying the ſubſtilar upon (or parallel to) the ſubſtilar before placed upon the plain, and ſo inſert all the houres from the paper to the plain.

After all this, you may make the ſtyle to the angle of the Poles elevation, and fit it in according to its requiſite place and poſition.

      ⊂ Note, that becauſe ſome of the houre points found in the Circle will happen ſo neere to the center of the Diall that you cannot well draw the houre-lines true; you may therefore help your ſelfe by that direction which I have given in my Geometricall way.

      [" This Geometricall way ſhall ſhortly be pub-" liſhed by the Authors own copie, with his own " Demonſtrations of the whole work.]

For drawing houres upon plains that have ſmall elevations, and upon Polar plains, uſe the former directions.

CHAP

CHAP. XXII.

A third way for all re-in-clining Dials.

Sect. I.

*To finde a re-in-clining plains diffe-
rence of Longitude from the South
part of your Meridian : and how
much the plains Meridian or (sub-
style) must lie from the Verticall line
of the plain.*

	I.	II.	III.
Complement of your Latit.	38 30 K	38 30 K	38 30 K
Plains verticall distance.	100 00 Z	60 00 Z	30 00 Z
Their Summe.	138 30	98 30	68 30
Their Difference.	61 30	21 30	8 30
Their halfe Summe.	69.15 R compl. 20 45 V	49.15 R comp. 40.45 V	34.15 R compl.55.45 V
Their halfe difference.	30.45 S.compl. 59.15 X	10 45 S compl. 79.15 X	4.15 S compl.85.45.X
Plains declinat. from Sou.	50 00 Y	140 00 Y	160 00 Y

Describe a Circle with your common (or lesser)
Scale of Chords.

L   And

And out of the same Scale make A Y =plains declination from South.

Out of the line ☉ make A R = R,
and A S = S, & draw YRM and M S B
and make A D = A B.

Out of the same line ☉ make A V = **V**,
and A X = X, and draw Y V N, and N X C.

☽ Then if K be lesse than Z {
CAD is the differ. of Longitude required.
and
C B is the angle between the substile & the verticall line.

☽ But if K be greater than Z {
C A D is the forementioned angle.
and
C B is the difference of Longitude.

These two arks **C** D and C B, must be measured from 90 in the line of Versed Sines, and looke what number of degrees they there cut, the same must be accounted for their quantities.

To finde the elevation of the Pole above the plain: and which of the Poles it is, whether North or South, that is elevated.

MEasure A B upon the Versed Sines (from 90) as before: the complement of that is E B. Measure also E C upon the same Scale, in the same manner. Count these quantities E B and E C (so found) upon the line ☉, and set them from E, to F and G, and make E r = R (taking E r out of the Scale of Versed Sines from 90) Draw r F O, and O G P. Measure E P upon your Scale of Chords, it will there give you the polar distance.
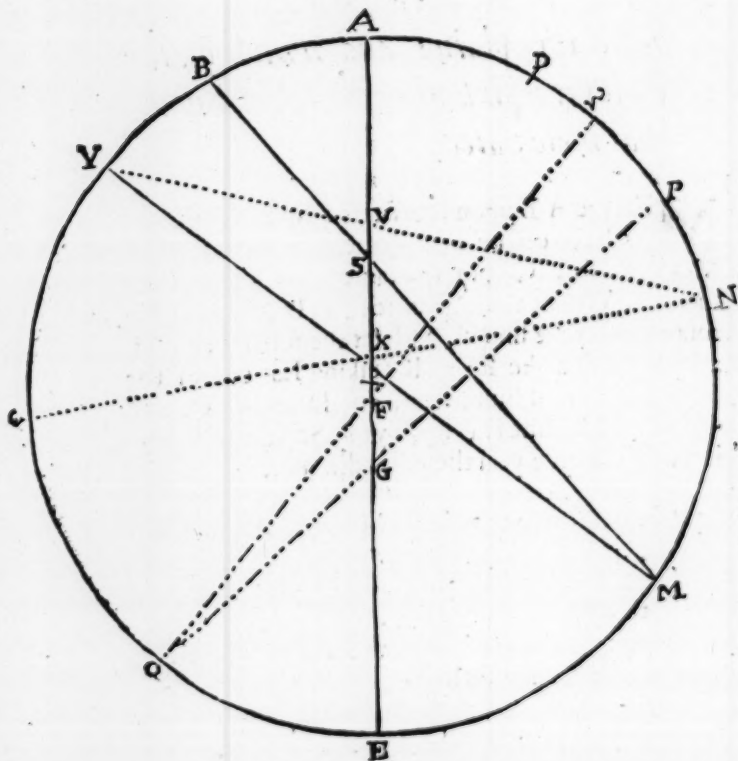
If E P fall to be 90, it is a Meridionall plain, and hath no Pole elevated.

If it be lesse than 90, the complement of it is the elevation of the North Pole.

If it be greater than 90. the excesse is the elevation of the South Pole.

❡ Note, that the three figures following have relation to the three Columns of the foregoing Table, and to these rules last delivered.

Figure I.

CAD	62	49
In this first Figure C B	34	13
EP	122	00

Figure II.

In this second Figure {	CAD	110	29
	CB	42	19
	EP	36	28

In

Figure III.

In this third Figure $\begin{cases} CAD & 118 \quad 36 \\ CB & 44 \quad 56 \\ EP & 14 \quad 00 \end{cases}$

S E C T.

Sect. 3.

Which way must the Substilar lie from the Verticall line?

THe Rules are the same with those before in the second way of Dialling, where the same Question is propounded. You may therefore have recourse to them. Or thus.

Upon all plains whereon the

North pole	South pole

is elevated; the substilar must lie from the

upper end	lower end

of the Verticall line towards the full

North.	South.

For drawing the houres, and finishing the Diall, you must doe as is prescribed in the 4th. *Sect.* of the former second way. For, having placed the Substilar, and knowing the plains difference of Longitude, you are to use the same course here that was there given.

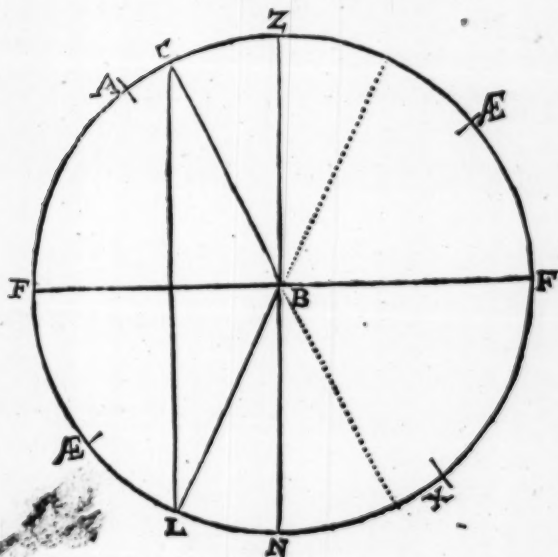It will be easie to doe these things in plains that are upright, and have no re-in-clination.

Note,

Note in all thefe three wayes of Dialling.

All directions here given fuppofe you to be in the
Northern Hemifphere of the world. If therefore
you fhould be in the Southern Hemifphere,
you may eafily make thefe precepts ferve
there too, by only altering the name
of North, Northen, &c. and
South Southern, &c.
one into the
other.

FINIS.

This Scheme hath relation to the 16th. Chapter, page 54.

Errata.

Page 32, in the Diagram, *for* H P G, *read* H F G.

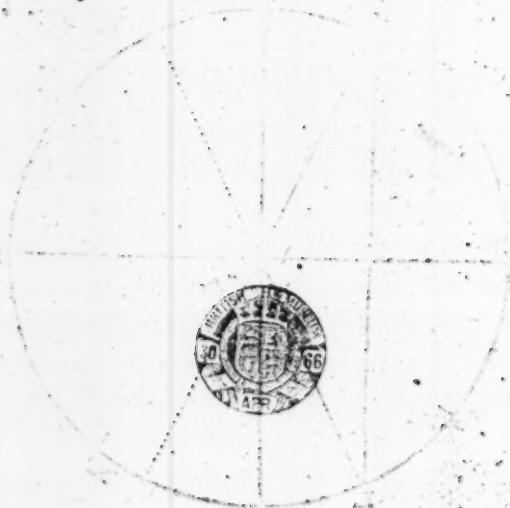Page 51, *for*, If D fall into E, *read*, If D fall into Æ.

Page 54, *line* 16, *for* For, *read* From.

In the Diall Page 57, at the other end of the line M, place the letter K upon the Substilar.

Page 62, line 19, *for* L P, *read* S P.

In the Diagram page 76, the letter R is wanting, at the intersection of the line Y M, with the line A E.